I

(1)

1). obtain the Fourier series for the function $f(x) = x \sin x$ in the interval $(0, 2\pi)$.

2). Find the fourier series of $f(x) = (\pi - x)^2$ in $(0, 2\pi)$. Deduce the sum of the series

i). $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$

(ii). $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots$

(3). obtain the Fourier series to represent the function $f(x) = |\cos x|, -\pi < x < \pi$.

(4). obtain the Fourier series $f(x) = \begin{cases} l - x; & 0 < x < l \\ 0; & l < x < 2l \end{cases}$

and hence deduce the following

(i) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$

(ii). $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$

(5). Find the Fourier series for the function $f(x) = \begin{cases} 2, & -2 < x < 0 \\ x; & 0 < x < 2 \end{cases}$. using the series, deduce the numerical series for π^2 and π.

(6). Find the half range fourier sine series of $f(x) = x(\pi - x)$ in $(0, \pi)$. Deduce that (ii). $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots = \frac{\pi^3}{32}$. (2)

(iii). $\frac{1}{1^6} + \frac{1}{3^6} + \frac{1}{5^6} + \cdots = \frac{\pi^6}{960}$.

(7). Find the half range cosine series for the function $f(x) = \begin{cases} x^2 ; & 0 < x < 1 \\ 2-x ; & 1 < x < 2 \end{cases}$

(8). Find the complex form of the Fourier series of $f(x) = e^{ax}$, $-l < x < l$.

(9). Find the Fourier series upto 2nd harmonic for $y = f(x)$ in $(0, 2\pi)$ for the data:

x:	0	1	2	3	4	5
y:	9	18	24	28	26	20

(10). The values of x and the corresponding values of $f(x)$ over a period T are given below. S.T $f(x) = 0.75 + 0.37 \cos\theta + 1.004 \sin\theta$, where $\theta = \frac{2\pi x}{T}$

x:	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
$f(x)$:	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

II

Find the Fourier transform of $f(x) = \begin{cases} a-|x| ; & |x|<a \\ 0 ; & |x|>0 \end{cases}$ and hence evaluate

(i). $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$

(ii). $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}$.

2). Show that the Fourier transform of

$f(x) = \begin{cases} a^2-x^2, & |x|\leq a \\ 0 ; & |x|>a<0 \end{cases}$ is $\frac{2\sqrt{2}}{\sqrt{\pi}}\left[\frac{\sin as - as\cos as}{s^3}\right]$

Deduce that (i). $\int_0^\infty \left(\frac{\sin t - t\cos t}{t^3}\right) dt = \frac{\pi}{4}$

(ii). $\int_0^\infty \left(\frac{\sin t - \cos t}{t^3}\right)^2 dt = \frac{\pi}{15}$

(iii) $\int_0^\infty \left(\frac{\sin x - x\cos x}{x^3}\right) \cos\frac{x}{2}\, dt = \frac{3\pi}{16}$

3. Show that $e^{-x^2/2}$ is self-reciprocal with respect to Fourier transform.

4. State and prove Convolution theorem on Fourier transform.

5. Derive the Parseval's Identity for Fourier transf

(6). Evaluate $\int_0^\infty \frac{x^2\,dx}{(a^2+x^2)(b^2+x^2)}$ if $a,b>0$ using Parseval's Identity.

(7). Evaluate $\int_0^\infty \frac{dx}{(a^2+x^2)^2}$ if $a>0$ using P.I.

(8). Find the Fourier cosine transform of $e^{-a^2x^2}$, $a>0$ and hence deduce that sine transform of $x\,e^{-a^2x^2}$.

(9). S.T the Fourier sine transform of $x\,e^{-x^2/2}$ is self reciprocal.

(10). Find the Fourier cosine and sine transform of $f(x)=\frac{e^{-ax}}{x}$.

III

(5)

1. Solve: $(D^2+2DD'+D'^2-2D-2D')z=\sin(x+2y)$

2. Solve: $r+s-6t=y\cos x$.

3. Solve: $p^2y(1+x^2)=qx^2$.

4. Solve: $q(p^2z+q^2)=4$.

5. Solve $z=px+qy+\sqrt{1+p^2+q^2}$

6. Solve: $x(z^2-y^2)p+y(x^2-z^2)q=z(y^2-x^2)$.

7. Solve: $(mz-ny)p+(nx-lz)q=(ly-mx)$.

8. Solve: $(x^2-y^2-z^2)p+2xyq-2xz=0$.

9. Form the pde by eliminating the arbitrary functions from $z=f(x+ct)+\phi(x-ct)$

10. Form the partial differential equation from $\phi(x^2+y^2+z^2,\ lx+my+nz)=0$.

IV

1. ~~note~~ A string is tightly stretched and its ends are fastened at two points $x=0$ and $x=l$. The mid point of the string is displaced transversely through a distance b and it is released from rest. Find $y(x,t)$.

② . The point of trisection of the string are pulled aside through a distance 'b' on the opposite side of the position of equilibrium and string is released from rest. Find an expression for displacement of string at any time.

③ . A string of length 'l' is initially at rest in its equilibrium position and each of its points is given a velocity $\frac{\partial y(x,0)}{\partial t} = V_0 \sin^3 \frac{\pi x}{l}$. $0 \le x \le l$. Determine $y(x,t)$.

④ . A rod of length 'l' is kept at temperature 0°C and 100°C at the ends A and B, untill steady state conditions prevail then the temperature at the end B is reduced to 0°C, while at A is maintained so. Find the temperature distribution along the rod.

⑤ . The end A and B of the rod 'l' cm have their temperature kept at 40°C and 90°C until steady state conditions prevail. The temperature at A is suddenly raised to 90°C and the same time B is suddenly lowered to 40°C. Find the temperature displacement $u(x,t)$.

V

using convolution theorem find $z^{-1}\left[\frac{8z^2}{(2z-1)(4z-1)}\right]$

(2). using convolution theorem find $z^{-1}\left[\frac{z^2}{(z+a)^2}\right]$

(3). using convolution theorem find $z^{-1}\left[\left(\frac{z}{z-4}\right)^3\right]$

(4). using Residue method find $z^{-1}\left[\frac{z(z+1)}{(z-1)^3}\right]$

(5). using Residue method find $z^{-1}\left[\frac{z^2-3z}{(z-5)(z+2)}\right]$

(6). using partial fraction method find

$z^{-1}\left[\frac{z(z^2-z+2)}{(z+1)(z-1)}\right]$

(7). Find the difference equation from $y_n = (A+Bn)2^n$.

(8). solve: $y(n+3) - 3y(n+1) + 2y(n) = 0$

$y(0)=4$, $y(1)=0$ and $y(2)=8$.

(9). solve: $y_{n+2} - 3y_{n+1} + 2y_n = 2^n$ with $y_0 = y_1 = 0$

(10). solve: $y(k+2) - 4y(k+1) + 4y(k) = 0$

where $y(0) = y(1) = 0$.