Performance of Direct Sequence Spread Spectrum System:

1. Processing gain.

Processing gain (PG) is defined as the ratio of the bandwidth of spread message signal to the bandwidth of unspreaded data signal.

$$\text{processing gain} = \frac{\text{Bw (spreading signal)}}{\text{Bw unspreaded signal.}}$$

For NRZ bipolar signals the Bw of the signal is equal to $\frac{1}{\text{one bit period}}$

Bw of the data signal $\text{Bw (data signal)} = \frac{1}{\text{one bit period}} = \frac{1}{T_b}$ → ①

$\text{Bw (spreaded message signal)} = \frac{1}{\text{one bit period}} = \frac{1}{T_c}$ → ②

$$\boxed{\therefore \text{processing Gain} = \frac{1/T_c}{1/T_b} = \frac{T_b}{T_c}}$$

One bit period 'T_b' of data signal is equal to 'N' bits periods of spreading pseudo-noise signal

$$T_b = NT_c$$

$$N = \frac{T_b}{T_c}.$$

probability of Error of DS/BPSK System:

$$p(e) = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_0}}$$

Here $N_0/2$ is a noise spectral density and E_b is the bit energy.

for direct sequence spread spectrum modulation the noise spectral density is given.

$$\frac{N_0}{2} = JT_c/2$$

$$N_0 = JT_c$$

J is the average interference power.

$$\boxed{P(e) = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{JT_c}}}$$

Jamming Margin (Antijam characteristics).

E_b is the bit energy. It is equal to $P_S T_b$

Here P_S — average signal power

T_b — one bit period.

Consider the ratio of signal Energy per bit to noise spectral density (E_b/N_0)

$$\frac{E_b}{N_0} = \frac{P_S T_b}{N_0}$$

$$N_0 = J T_c$$

$$\frac{E_b}{N_0} = \frac{P_S T_b}{J T_c}$$

$$\frac{E_b}{N_0} = \left(\frac{P_S}{J}\right)\left(\frac{T_b}{T_c}\right)$$

$$\frac{J}{P_S} = \frac{T_b / T_c}{E_b / N_0} = \frac{PG}{E_b/N_0}.$$

Jamming Margin:

P_S - data signal

$$\left(\frac{J}{P_S}\right) = (PG)_{dB} - \left(\frac{E_b}{N_0}\right)_{dB}.$$

$$(\text{Jamming Margin})_{dB} = (\text{processing Gain})_{dB} - 10\log_{10}\left(\frac{E_b}{N_0}\right)$$

b. The direct sequence spread spectrum communication system has following parameters:
Data sequence bit duration $T_b = 4.095$ ms
PN chip duration $T_c = 1 \mu s$
$E_b/N_0 = 10$ for average probability of error less than 10^{-5}
Calculate processing gain and jamming margin.

1. $PG = \frac{T_b}{T_c} = \frac{4.095 \times 10^{-3}}{1 \times 10^{-6}} = 4095$

PG=N the Length of the bit sequence is 4095.

Jamming Margin $= \frac{J}{P_S} = \frac{PG}{E_b/N_0}$

$= \frac{4095}{10} = 409.5$

$\left(\frac{J}{P_S}\right)_{dB} = PG_{dB} - 10 \log_{10}\left(\frac{E_b}{N_0}\right)$

$= 10 \log_{10}(4095) - 10 \log_{10}(10)$

$= 36.1 - 10 = 26.1$ dB.

properties of Max Length Sequence:

1. Balance property:

The number of 1's always one more than the number of zeros in each period of a max length sequence. where as in truly random binary sequence 1's and 0's equally probable.

Ex:

The PN Sequence:

$c_n = 0011101$

here 3 zero 4 one's. no of One's > 1, zero's.

so It is prove Balance property.

Run property:

The run means subsequence of identical symbols 1's or 0's within one period of the sequence.

The length of the run is equal to length of the sub sequence. When the max length sequence is generated by feedback shift reg of length m. then the total number of runs is 2^{m-1}

Ex: $m = 3$. $2^{m-1} = 4$ runs:

$c_n = \underbrace{00}_{\text{Run } 1}\ \underbrace{111}_{2}\ \underbrace{0}_{3}\ \underbrace{1}_{4}$

half of the total run is length one.
1/4 of the total run is length two.
1/8 of the total run is length three.

Run 1 = {00} length 1 : run 2 and 4
Run 2 = {111} length 2 : run 1.
3 = {0}.
4 = {1}

3. Correlation property:

The auto correlation function of max length sequence is periodic & it is binary valued, Let N represents the period.

$$N = 2^{m} - 1$$

m is Length of the feedback Shift reg:

T_c bits of max Length Sequence $= \frac{1}{R_c}$

R_c – chip rate.

$$\boxed{T_b = N T_c}$$

N – Length of the Sequence.

Autocorrelation function of PN Sequence c(t)

$$R_c(\tau) = \frac{1}{T_b} \int_{-T_b/2}^{T_b/2} c(t)\, c(t-\tau)\, dt.$$

for the PN-Sequence. c(t)

$$R_c(\tau) = \begin{cases} 1 - \frac{N+1}{NT_c} |\tau| & \text{for } |\tau| < T_c \\ -\frac{1}{N} & \text{else where.} \end{cases}$$

The fourier Transform of the auto Correlation function gives power spectral density:

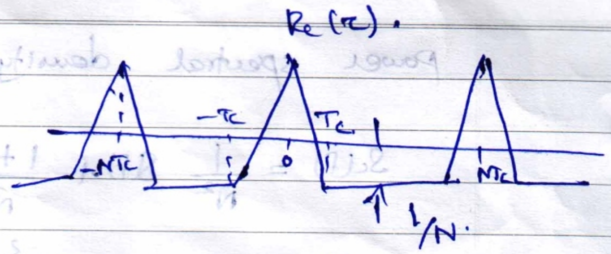
$$S_c(f) = FT\{R_c(\tau)\}$$

$$= \frac{1}{N^2} \delta(f) + \frac{1+N}{N^2} \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \operatorname{sinc}^2\left(\frac{n}{N}\right) \delta\left(f - \frac{n}{NT_c}\right)$$

$m = 5$

$N = 2^m - 1 = 31.$

$R_c = 10\,KHz.$

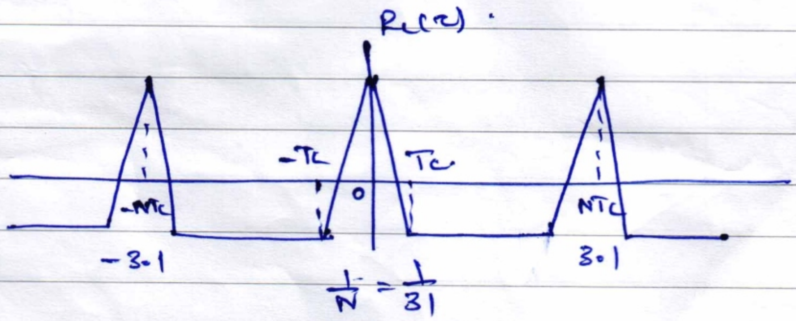


$T_c = \frac{1}{R_c} = \frac{1}{10000} = 0.1\ msec.$

$$R_c(\tau) = \begin{cases} 1 - \frac{N+1}{NT_c}|\tau| & |\tau| < T_c \\ -\frac{1}{N} & \text{other wise.} \end{cases}$$

$$R_c(\tau) = \begin{cases} 1 - \frac{31+1}{31 \times 0.1 \times 10^{-3}}|\tau| & |\tau| < 0.1\ msec. \\ -\frac{1}{31}. & \text{otherwise.} \end{cases}$$

$$= \begin{cases} 1 - (10322.6)\,|\tau| & |\tau| < 0.1\ msec. \\ -\frac{1}{31} & \text{elsewhere.} \end{cases}$$



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power spectral density of the sequence:

$$S_c(f) = \frac{1}{N^2}\,\delta(f) + \frac{1+N}{N^2} \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \left(\frac{n}{N}\right) \delta\left(f - \frac{n}{NT_c}\right)$$

$N = 31 \qquad T_c = 0.1\times 10^{-3}$

$$S_c(f) = \frac{1}{(31)^2}\,\delta(f) + \frac{1+31}{(31)^2} \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \operatorname{sinc}^2\left(\frac{n}{31}\right) \delta\left(f - \frac{n}{31\times 0.1\times 10^{-3}}\right)$$

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